Broadgate, London

Testing robustness of longevity models

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Overview

1. Robustness
2. Sample paths as a stability test
3. Other model assessments
4. Conclusions

1. What is robustness?

The absence of embarrassment

- The regulator requires you choose a model
- Next year —

. . . will it fit?

. . . will the forecast be consistent?

1. Forecasts in the face of change

What will happen next year?

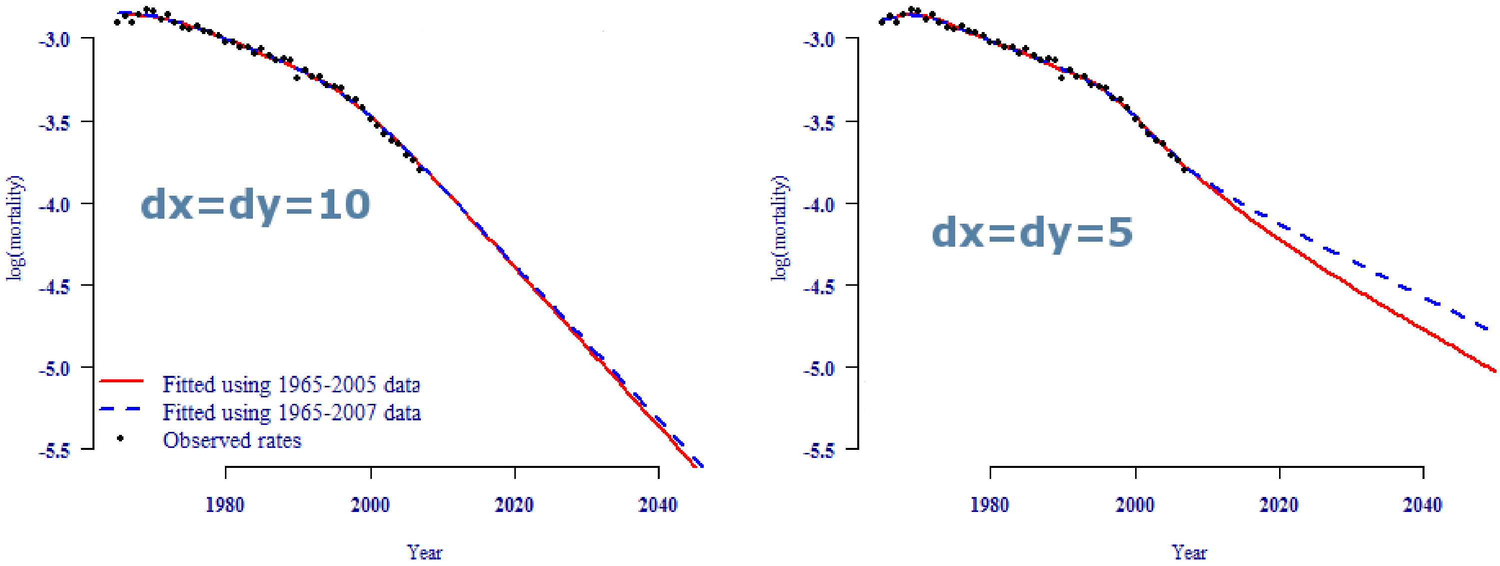
- Models *must* allow for uncertainty
- How will model forecasts respond to new data year-on-year?

"[...] the Working Party was concerned that adopting a P-spline methodology could introduce [an] unwanted "edge effect", i.e. that the initial rates of improvement for the final year of the data would be unduly influenced by the relative level of experience in that final year and hence would vary materially from one year to the next."

CMI Working Paper 39, page 4

1. Stability versus sensitivity

- Model fit does not necessarily provide forecast stability
- The best fitting model might be *too* responsive



Source: Observed, smoothed and projected mortality rates for EW males aged 70. ONS data for males aged 40–104 smoothed using 2D age-period model with dx=dy=5, dx=dy=10.

1. Factors influencing forecast stability

Model Type	Factors influencing stability
All models	Model definition
	Convergence criteria
	Dataset population
	Dataset age range
	Dataset period covered
	Weighting/outliers

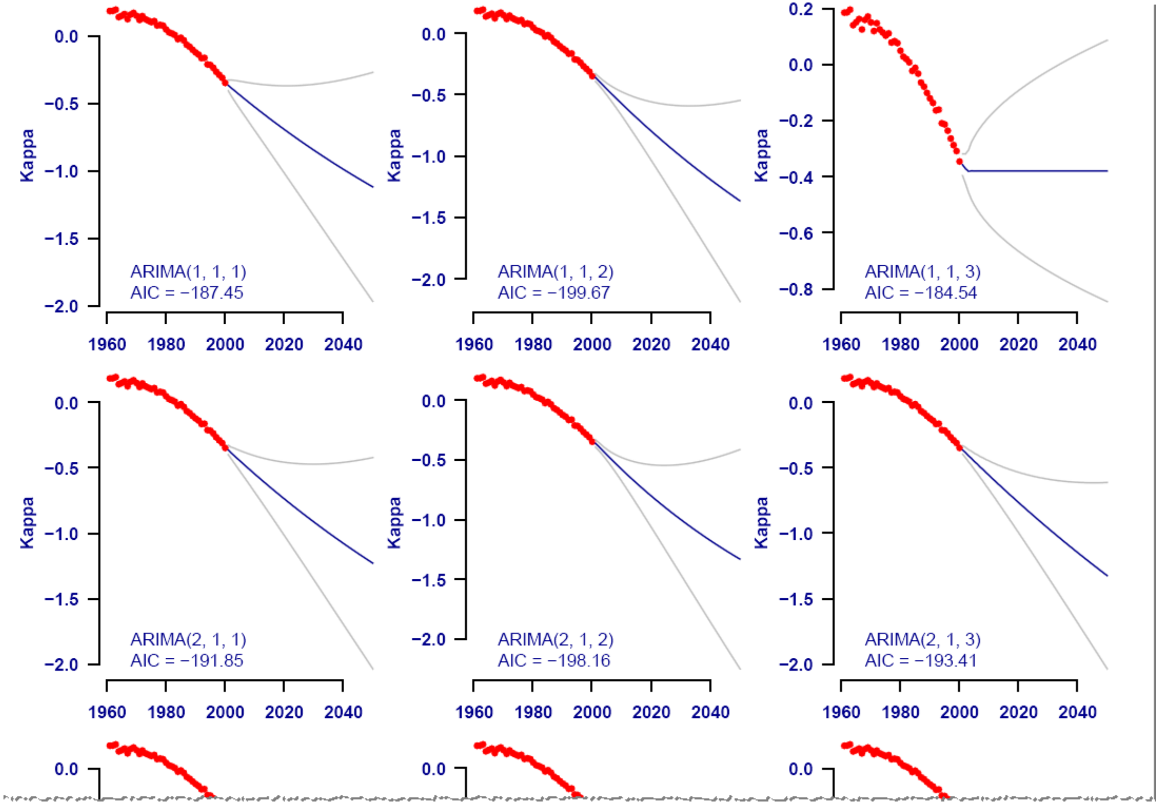
1. Factors influencing forecast stability

Model Type	Factors influencing stability
Models using penalties	Knot spacing Knot positioning Overdispersion Effective dimension/degrees of freedom

1. Factors influencing forecast stability

Model Type	**Factors influencing stability**
Time-series models	Choice of (p,d,q) values Choice of drift model type ARIMA search space

1. Factors influencing forecast stability



Source: Projection Toolkit: Lee Carter DDE model against ONS EW Males 1961–2000 data for males aged 50–80 years.

. . . So how can we know if a model will let us down?

2. Sample paths as built-in stability test

or, model test thyself...

2. What are sample paths?

- A sample path is one possible simulation of future mortality

  … essentially a grid of mortality rates by age and year
- All stochastic models can potentially be used to generate sample paths
- Paths can incorporate trend (parameter) uncertainty and/or volatility

  … by resampling residuals (Penalty models)

  … by applying noise variance (ARIMA models)

2. A basis for stability testing

Stability testing - the question we ask is important:

- How will our model fare
- . . . in the future?
- . . . in response to its own sample paths?

2. A basis for stability testing

Stability testing — the question we ask is important:

- How will our model fare
- . . . in the future? — Unknowable
- . . . in response to its own sample paths? — Can be tested!

2. Simulating the next years experience

Assume no (or balanced) migration affecting population

- From population: deaths and exposures for $year_y$
- ... and hence $initial\ exposures_{y+1}$
- From sample path: $mortality\ rates_{y+1}$
- Simulate $deaths_{y+1}$ by applying $mortality\ rates_{y+1}$ to $exposures_{y+1}$
- ... Age population by one year and apply forecast deaths

2. Stability key questions

- Each projection is a simulation we can use to assess stability
- Do any of our model fits fail?
- What warnings do our model fits raise?
- How stable/volatile are model metrics (e.g. annuity values) by age?:

... The coefficient of variation (CV) is defined as a

... ratio of the standard deviation σ to the mean, μ:

$$c_v = \frac{\sigma}{\mu}$$

2. Assessing stability

	A	B	C
5	**2. Inputs**		
6	LiabilityType	Annuity factor	
7	Age	60	
8	Percentile	99.5%	
9	Methodology	Relative to average	
10	Baseline	17.15	
11	Average	17.36	
12	Variance	1.22	
13	Standard deviation	1.11	
14	Coefficient of variation	6.4%	Variability suggests instability in underlying projection model
15	99.5th percentile	20.30	
16	Sample quantile capital requirement	16.97%	
17	Skewness	-0.19	
18	Kurtosis	5.07	
19	Capital requirement assuming Normality	16.42%	*only valid if Normally distributed, e.g. if skewness is zero and kurtosis is 3.*

Source: Projection Toolkit: 500 simulation stability test for Lee-Carter Currie-Richards model(no overdispersion, dx=dy=5) against ONS EW Males (1961–2010, 50–104). Assessing age 60 using a 5pct stability warning threshold.

3. Other model assessments

Other means of sense-checking models are valuable

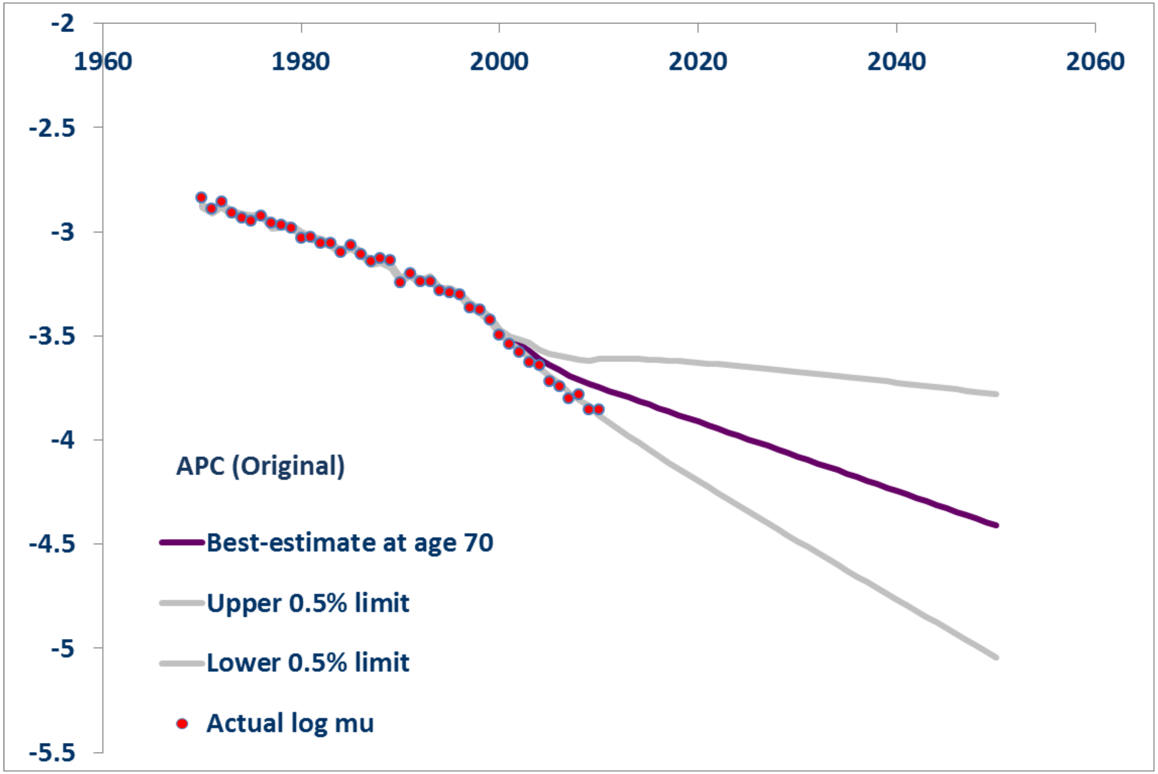
- Backtesting — How would our model fare in the past?
- Benchmarking — How does our model compare to other projections?

3. Other model assessments - Backtesting

How does our model stand up to past experience?

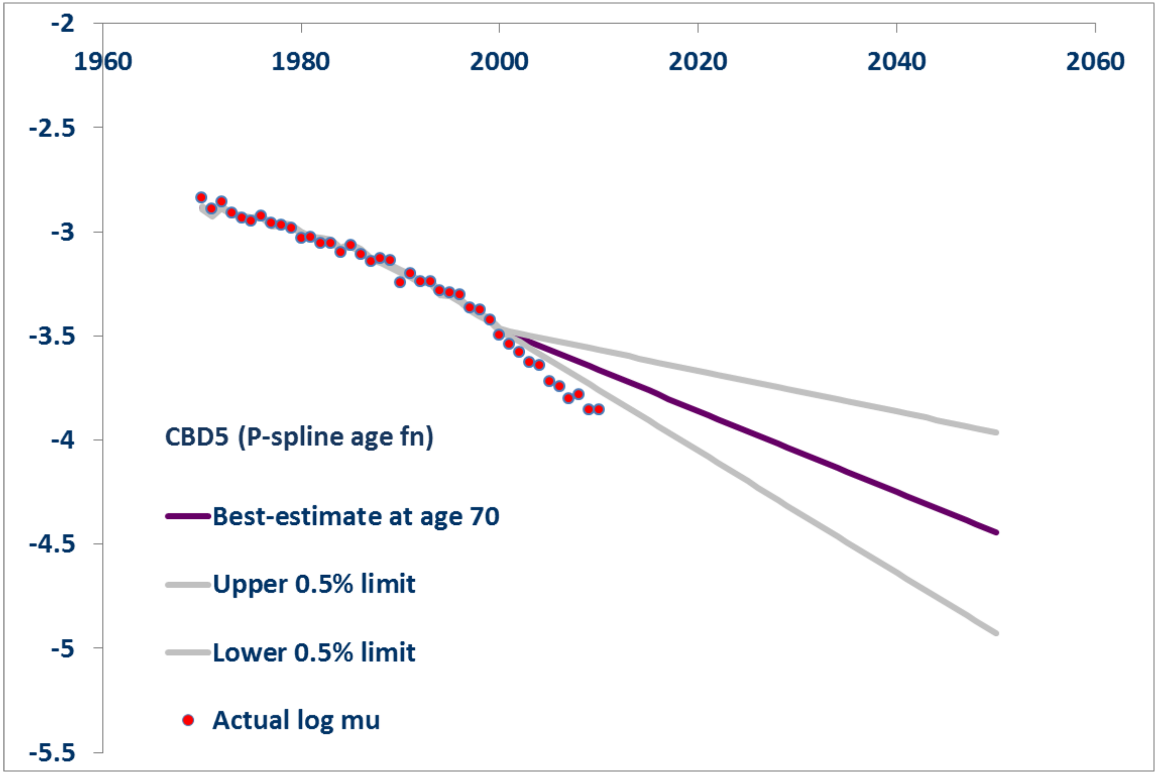
- A visual approach — fit a model to a historic range
- Plot forecast rates plus trend uncertainty
- Add observed rates — do they sit in the funnel?

3. Backtest - APC



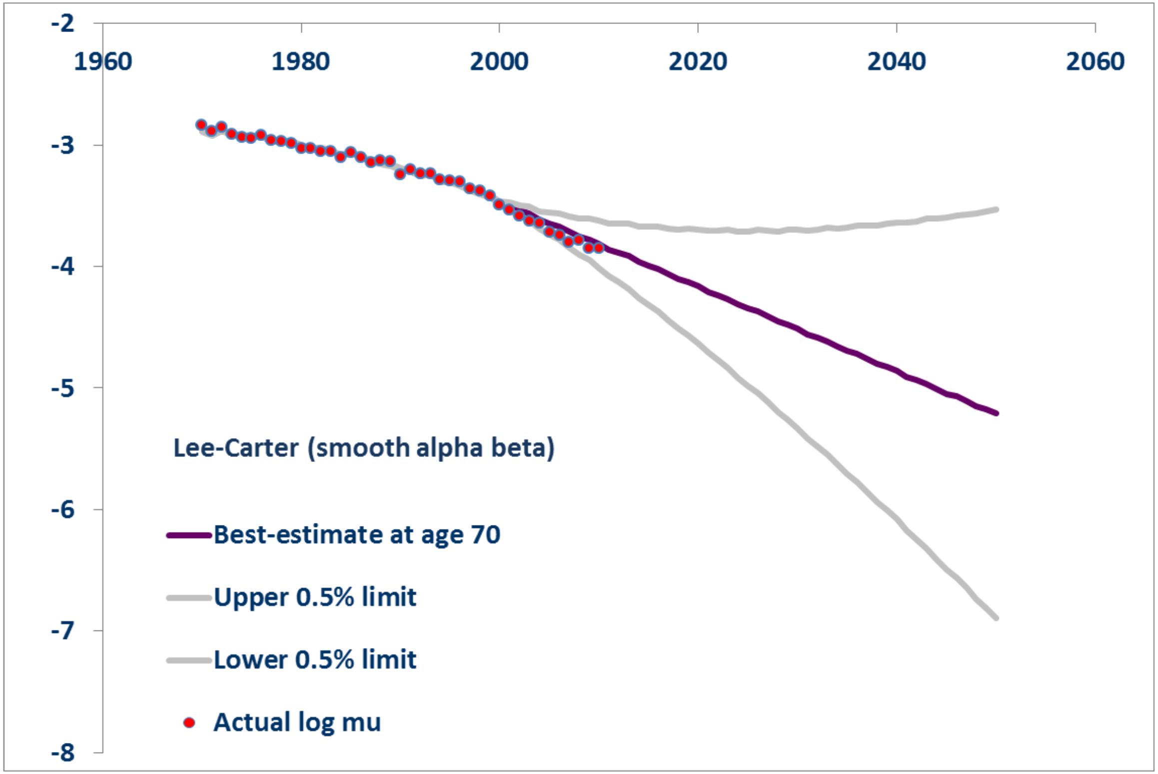
Source: Projection Toolkit: APC (original) model against ONS EW Males 1970–2000 data for males aged 60–90 years.

3. Backtest - CBD5



Source: Projection Toolkit: CBD5 model against ONS EW Males 1970–2000 data for males aged 60–90 years.

3. Backtest - LC Smooth alpha beta



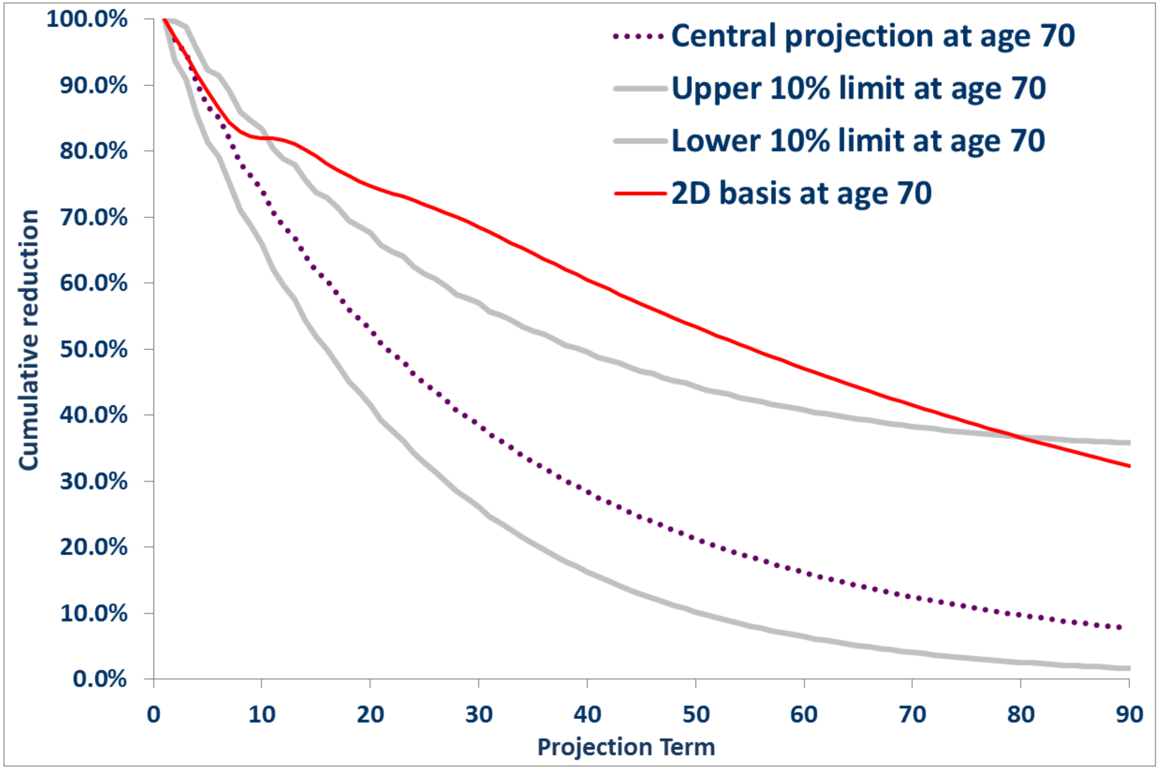
Source: Projection Toolkit: Lee-Carter smooth alpha and beta model against ONS EW Males 1970–2000 data for males aged 60–90 years.

3. Other model assessments - Benchmarking

Are bases in support or in conflict?

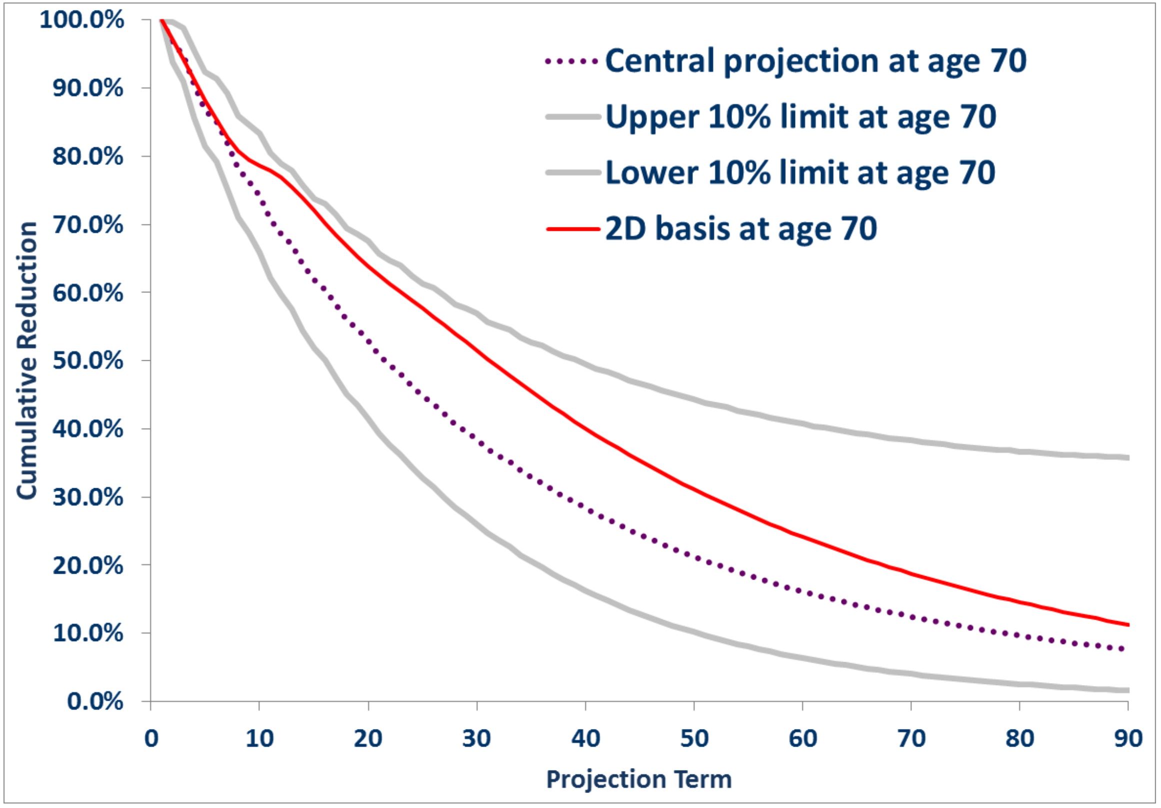
- A visual approach — plot forecast reduction factors
- Add trend uncertainty bounds
- Plot alternative basis (e.g COD, or CMI expectations)

3. Benchmark - LC Smooth alpha beta



Source: Projection Toolkit: CMI 2010 1.25pct against Lee-Carter smooth alpha and beta model against ONS EW Males 1961–2010 data for males aged 50–95 years.

3. Benchmark - LC Smooth alpha beta



Source: Projection Toolkit: CMI 2010 2.5pct against Lee-Carter smooth alpha and beta model against ONS EW Males 1961–2010 data for males aged 50–95 years.

4. Conclusions

We don't know the future — but we do know our model forecast

We should expect a robust and stable model:

- . . . not to fail in response to reasonable new data
- . . . to show acceptable volatility in response to its own paths

Robustness and stability are key aspects of model risk

References

Richards, S. J., Currie, I. D. and Ritchie, G. P. **2012** *A value-at-risk framework for longevity trend risk*, Longevitas Ltd

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